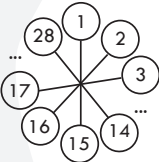


2022 Grade Five Spirit of Math Contest Solutions

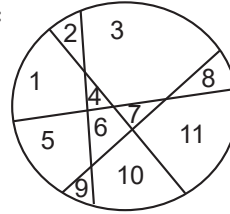
- 1) **B** Follow the Order of Operations and calculate: $30 \div 6 - 10 + 8 = 5 - 10 + 8 = 3$
- 2) **C** Since one quarter is equal to one fourth, Carson ate $400 \div 4 = 100$ g of pizza.
- 3) **A** To find the unit's digit of the product, you only need to multiply the unit's digits. Since $7 \times 6 = 42$, the unit's digit of 87×46 is 2.
- 4) **C** Any whole number multiplied by itself has a product that is a perfect square. Of the options given, only $6 \times 6 = 36$ is a perfect square.
- 5) **D** There are 50 minutes from 3:10 p.m. to 4:00 p.m. and another 5 minutes from 4:00 p.m. to 4:05 p.m. Altogether, Marcel volunteered for $50 + 5 = 55$ minutes.
- 6) **D** There are 16 small shaded squares. If each of the small squares has an area 2 cm^2 then the area of shaded region is $16 \times 2 = 32 \text{ cm}^2$.
- 7) **B** The numbers 2, 12, and 20 contain the digit 2. Therefore, the printer printed three 2's.
- 8) **B** If Stella is nine years old next year, Lucy will be $9 + 5 = 14$ years old next year. Therefore, Lucy is $14 - 1 = 13$ years old this year.
- 9) **A** Since each row, column, and diagonal have the same sum, they each add up to $9 + 8 + 7 = 24$. The missing number in the bottom left corner is $24 - 6 - 7 = 11$ and x is $24 - 11 - 8 = 5$. There are multiple ways to find the answer.
- | | | |
|----|----|----|
| 9 | 10 | 5 |
| 4 | 8 | 12 |
| 11 | 6 | 7 |
- 10) **C** There are 4 choices for the first letter, and for each of these choices, there are 3 choices for the second letter, and for each of these choices, there are 2 choices for the third letter and 1 choice for the fourth letter. Altogether, there are $4 \times 3 \times 2 \times 1 = 24$ arrangements.
- 11) **B** There are a total of $6 + 1 + 3 = 10$ balls in the bag. Three of the ten balls are blue, so the probability of picking a blue ball is $\frac{3}{10}$.
- 12) **B** Since $3^2 = 9$ is the first perfect square greater than 5 and $7^2 = 49$ is the last perfect square before 50, all perfect squares from 3^2 to 7^2 are between 5 and 50. There are $7 - 2 = 5$ perfect squares.
- 13) **C** Cynthia did not study the 24 pages before page 25. Therefore, she studied $32 - 24 = 8$ pages.
- 14) **B** Write the tablespoons of sugar as fractions and add: $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$
- 15) **D** The factors of 18 are $\{1, 2, 3, 6, 9, 18\}$. Therefore, 18 has 6 factors.
- 16) **D** Eddie played a total of $3 + 4 + 5 + 4 + 2 = 18$ hours outside.
- 17) **A** The sum of durians and papayas is $12 + 6 = 18$. The sum of star fruits and rambutans is $8 + 24 = 32$. Since 18 and 32 have a common factor of 2, the ratio 18:32 can be simplified to $18 \div 2 = 9$ to $32 \div 2 = 16$ or 9:16.
- 18) **D** The sum of two odd numbers or two even numbers is always even. Since $9 \div 2 = 4 \text{ R } 1$, there can be at most 4 pairs of odd numbers, or $4 \times 2 = 8$ odd numbers added together to sum to an even number. The 9th number must be even and when added with the first 8 numbers it produces an even number.
- 19) **B** The amount each term is increasing by follows a pattern: $17 - 14 = 3$, $23 - 17 = 6$, $32 - 23 = 9$, and $44 - 32 = 12$. Since this pattern is increasing by 3, the next term will be $12 + 3 = 15$ larger than 44. Therefore, the next term in the sequence is $44 + 15 = 59$.
- 20) **A** There are $28 \div 2 = 14$ horses in half the carousel. Naomi is 14 horses in front of Sanjay, thus she is on horse $3 + 14 = 17$.
- 
- 21) **D** The sum of angles in a triangle is 180° so the sum of remaining angles is $180 - 45 = 135^\circ$. The sum of angles in a quadrilateral is 360° so the sum of remaining angles is $360 - 45 = 315^\circ$. The difference is $315 - 135 = 180^\circ$.



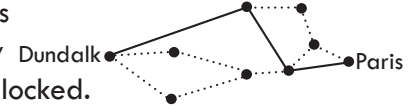
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- 22) **C** Draw the circle and the chords and look for a pattern:

# Chords	Regions	Pattern
0	1	1
1	2	$1 + 1 = 2$
2	4	$1 + 1 + 2 = 4$
3	7	$1 + 1 + 2 + 3 = 7$
4	11	$1 + 1 + 2 + 3 + 4 = 11$



Therefore, four chords divide the circle into at most 11 regions.

- 23) **C** The path with the fewest streams from Dundalk to Paris contains 3 streams. If all other streams are removed, water will still flow from Dundalk to Paris. Therefore, $12 - 3 = 9$ streams can be blocked.
- 
- 24) **A** The nine classmates take $5 \times 9 = 45$ minutes to complete the drill in total. With Tabitha's time, the ten students take a total of $45 + 10 = 55$ minutes to complete the drill. The average time it takes a student to complete the drill is $55 \div 10 = 5.5$ minutes.
- 25) **C** S is a carry over into the hundreds column, so $S = 1$. Since a carry over from the $O + O$ in the ones column results in the units digit of $O + O$ to be O , $O + O$ must have a units digit that is one less than O . This is only possible if $O = 9$. In the ones column, $9 + 9 = 18$, so $M = 8$. The sum of the digits of SOM is therefore $1 + 9 + 8 = 18$.
- 26) **B** $a \# b$ is defined as $a^b - b \times a$. Insert $a = 6$ and $b = 3$ to get:
- $$6 \# 3 = 6^3 - 3 \times 6$$
- $$= 216 - 18$$
- $$= 198$$
- 27) **B** If the oven can bake eight pizzas per half hour, then it can bake $8 \times 2 = 16$ pizzas per hour. In six hours, that is $16 \times 6 = 96$ pizzas baked. The customers bought 24 out of 96 pizzas, which can be reduced to $\frac{1}{4}$ which is 25%.
- 28) **B** To make six shorter clips, Millie needs to make five cuts. If it takes one minute to make five cuts, it takes $60 \div 5 = 12$ seconds to make one cut. To make 10 shorter clips, Millie has to make nine cuts. This will take her $9 \times 12 = 108$ seconds.
- 29) **D** The prime factorization of 1890 is $2 \times 3 \times 3 \times 3 \times 5 \times 7$. The numbers 5 and 7 cannot be multiplied with other numbers or they would no longer be single digit numbers. The only way to have two other single digits is if they are $2 \times 3 = 6$ and $3 \times 3 = 9$. So the sum of the digits is $5 + 7 + 6 + 9 = 27$.
- 30) **A** The 4-digit numbers are from 1001 to 9999. For a 4-digit number to be a palindrome, the first two digits should mirror the last two digits. The first two digits can be any number from 10 to 99. There are $99 - 9 = 90$ numbers from 10 to 99 so there are 90 4-digit palindromes.
- 31) **B** The bell rings every 1, 2, 3, ..., 7 seconds so the next time they will ring together will be a multiple of 1, 2, 3, ..., and 7. Prime factor 1, 2, 3, ..., 7 to find their least common multiple, as shown to the right. The least common multiple is 420 seconds, which is equal to $420 \div 60 = 7$ minutes. They will ring together again 7 minutes after 12:00 p.m., which is 12:07 p.m.
- | | | | | |
|----------------------------------|---|----------------|---|---|
| 2 | = | 2 | | |
| 3 | = | | 3 | |
| 4 | = | 2 ² | | |
| 5 | = | | | 5 |
| 6 | = | 2 | × | 3 |
| 7 | = | | | 7 |
| LCM = 2 ² × 3 × 5 × 7 | | | | |
| = 420 | | | | |



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- 32) **D** Since all statements contradict each other, only one can be true. Reword each statement to reflect which other statements are true. Statement 1 says one statement is false, therefore four are true. This cannot be the case since there is only one true statement. Similarly, Statements 2 and 3 indicate more than one true statement. Statement 4 says four statements are false, therefore one is true. This is the case and Statement 4 is therefore the true statement. Statement 5 says all statements are false, which cannot be the case because Statement 4 is already proven to be true.
- 33) **D** If the length and width are each three times larger, then the area of the base has increased to be $3 \times 3 = 9$ times larger than the original dimensions. Therefore, it takes $9 \times 3 = 27$ minutes for Michael to fill his second fish tank.
- 34) **A** The greatest number of boxes they can fill must be a factor of both numbers. The prime factorization of 2970 is $2 \times 3^3 \times 5 \times 11$ and of 4410 is $2 \times 3^2 \times 5 \times 7^2$. The greatest common factor of 2970 and 4410 is $2 \times 3^2 \times 5 = 90$ so they can fill at most 90 boxes.
- 35) **B** Draw a Venn diagram. If seven students had all three toppings, then $16 - 7 = 9$ had only pearl and grass jelly, $15 - 7 = 8$ had only pearl and aloe, and $14 - 7 = 7$ had only aloe and grass jelly. So, $62 - 9 - 8 - 7 = 38$ had only pearl, $45 - 8 - 7 - 7 = 23$ had only aloe, and $42 - 9 - 7 - 7 = 19$ had only grass jelly. Therefore, $38 + 8 + 23 + 9 + 7 + 7 + 19 = 111$ cups of tea with toppings were sold.
- Total: 111

62 Pearl	45 Aloe
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42 Grass Jelly

- 36) **A** Find the volume of the container and subtract the volume of the three tennis balls. The radius of one ball is 4 cm, so the diameter of one ball is $4 \times 2 = 8$ cm and the height of the container is $3 \times 8 = 24$ cm. The volume of a cylinder is given by the formula $\pi r^2 h$, so the volume of the container is $\pi(4^2) \times (24) = 384\pi \text{ cm}^3$. The volume of a sphere is given by the formula $\frac{4}{3}\pi r^3$, so the volume of three spheres is $3 \times \frac{4}{3}\pi r^3 = 4\pi r^3$. Thus, the volume of the three tennis balls is $4\pi(4^3) = 256\pi \text{ cm}^3$. Therefore the volume in the container not occupied by the tennis balls is $384\pi - 256\pi = 128\pi \text{ cm}^3$.
- 37) **D** In the left column alone, one-third of the entire grid is shaded. A region one third of the size of this column, which is one-ninth of the entire grid, is shaded in the middle of the top row. A region one third of this, which is one-twenty-seventh of the entire grid, is shaded in the right column of the center square. This pattern repeats indefinitely such that half of the entire grid is shaded. Therefore, the diagram proves that: $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots = \frac{1}{2}$
- 38) **C** Review the given options to find one that is certainly true.
 Option A: False. If Yui barks, her neighbours will wake up.
 Option B: False. If Yui's neighbours wake up, Yui did not necessarily see a noisy motorcycle.
 Option C: True. If Yui's neighbours did not wake up, then Yui did not see a motorcycle.
 Option D: False. If Yui does not bark, her neighbours might still wake up.
- 39) **A** The Math Machine accepts multiples of 2 and 3, but not multiples of 6. So, add up all multiples of 2 and 3. However, in each of the multiples of 2 and 3, the multiples of 6 are counted, so subtract the multiples of 6 twice: $\frac{n(n+2)}{4} + \frac{n(n+3)}{6} - 2 \times \frac{n(n+6)}{12} = \frac{300 \times 302}{4} + \frac{300 \times 303}{6} - 2 \times \frac{300 \times 306}{12}$
- $$= 22650 + 15150 - 15300$$
- $$= 22500$$
- Therefore, the sum of the numbers the Math Machine accepts is 22 500.
- 40) **B** The sum of the top left corners is $6 + 5 + 6 = 17$. The sum of the top right corners is $1 + 9 + 7 = 17$. The sum of the bottom left corners is $8 + 4 + 5 = 17$. So, the sum of the corresponding corners is always 17. Therefore, the missing number is $17 - 3 - 9 = 5$.

