2022 Grade Six Spirit of Math Contest Solutions

1) **B** Using order of operations, calculate: $5 \times 9 - 10 + 2 = 45 - 10 + 2$
$= 37$

2) **A** To find the unit's digit of the product, you only need to multiply the unit's digits. Since $8 \times 4 = 32$, the unit's digit of 128×364 is 2.

3) **B** The pattern is to multiply the previous term by 2. The next term is $2 \times 24 = 48$.

4) **A** If Allen is 17 years old next year, then Barry will be $17 - 5 = 12$ years old next year. This year, Barry is $12 - 1 = 11$.

5) **C** A full circle is 360°. There are 4 equal slices, so the angle x in one slice is $360 \div 4 = 90°$.

6) **A** There is $55 \div 11 = 5$ times the salt, so there are $1 \times 5 = 5$ grams of pepper.

7) **B** The total percentage of those who did not order the mystery soup is $10\% + 15\% + 55\% = 80\%$. Therefore, $100\% - 80\% = 20\%$ of the people ordered the mystery soup.

8) **B** Since each row, column, and diagonal has a sum of 34, the missing number in the first column and first row is $34 - 14 - 15 - 1 = 4$ and the missing number in the last column and last row is $34 - 4 - 7 - 10 = 13$. Therefore, x is $34 - 1 - 12 - 13 = 8$.

4	9	5	16
14	7	11	2
15	6	10	3
1	12	8	13

9) **D** The factor set of 24 is {1, 2, 3, 4, 6, 8, 12, 24}. There are 8 numbers in this factor set.

10) **D** A regular pentagon has five equal sides so its perimeter must be divisible by 5. Of the options given, only 1035 is divisible by 5.

11) **C** There are $19 + 2 = 21$ cupcakes needed, and dividing them into packages of four requires $21 \div 4 = 5.25$ packages. Therefore, Anna should buy 6 packages of cupcakes.

12) **B** In total, Denton spent $3 + 4 + 2 + 5 + 1 = 15$ hours using his phone in five days.
That is an average of $15 \div 5 = 3$ hours per day.

13) **A** Since 25% of 12 is $0.25 \times 12 = 3$ and 25% of 16 is $0.25 \times 16 = 4$, you have $3 + 4 = 7$ fruits.

14) **A** Using order of operations, calculate: $a + b \times a^2 = 2 + 3 \times 2^2$
$= 2 + 3 \times 4$
$= 2 + 12$
$= 14$

15) **B** If all three numbers were the same, every number would be $333 \div 3 = 111$. However, the numbers are consecutive, so they must be split evenly on either side of 111 so that the sum remains the same. Therefore, the three numbers are 110, 111, and 112. The smallest number is 110.

16) **B** The sum of the number of woodwinds in both years is 34 and the difference between the number of woodwinds each year is 4. So, twice the number of woodwinds this year is $34 - 4 = 30$, thus there are $30 \div 2 = 15$ woodwinds in the band this year.

17) **B** If each brother can say he has three other brothers, then there must be four brothers in the family. The ratio is of brothers to parents is 4:2, which simplifies to 2:1.

18) **D** Check perfect squares with three digit squares to see which of the options is correct:
$16 + \sqrt{16} + 16^2 = 276$

$25 + \sqrt{25} + 25^2 = 655$
Of the options given, the sum is 655.

19) **C** The shaded area is equal to the area of the large circle subtract the area of the small circle. The area of a circle is πr^2. So, the area of the large circle is $\pi(4^2) = 16\pi$ cm^2 and the area of the small circle is $\pi(2^2) = 4\pi$ cm^2. Thus the shaded area is $16\pi - 4\pi = 12\pi$ cm^2.

20) **A** Each house is connected to seven other houses, so $8 \times 7 = 56$ wires. However, each wire has been counted twice. So there should be $56 \div 2 = 28$ wires. There are 11 wires shown in the diagram so $28 - 11 = 17$ wires broke in the storm.

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21) **C** To have the most remaining candies, many of the remaining candies should have small numbers. Begin by adding the smallest of the numbered candies: $1 + 2 + 3 + 4 + 5 = 15$. A total of $25 - 15 = 10$ is left to be added. The only way to get 10 from the remaining numbers is by using the candy with the number 10. Therefore, there are at most 6 remaining candies: 1, 2, 3, 4, 5, and 10. Note that there are many possible combinations of 6 remaining candies.

22) **B** Check each option: Zane has at least $1 + 1 + 1 + 1 = 4$® so he cannot have 3®. He could have $32 + 8 + 4 + 1 = 45$®. To have 47®, he must have one 32® and one 8® leaving $47 - 32 - 8 = 7$® remaining. He cannot create 7® with two coins so he must not have 47®. To have 63®, he must have one 32® and one 16® leaving $63 - 32 - 16 = 15$® remaining. He cannot create 15® with two coins so he must not have 63®.

23) **D** Each of the nine flowers has at least four petals, so $9 \times 4 = 36$ petals are accounted for. There are $39 - 36 = 3$ petals left that belong to yellow flowers. Each yellow flower needs one more petal added to the four already there, so there are 3 yellow flowers and $9 - 3 = 6$ red flowers.

24) **A** There are 7! ways to arrange 7 books on a shelf. Divide by 4! for the arrangements of the red books and 2! for the green books.

$$\frac{7!}{4!\,2!} = \frac{7\times6\times5\times4\times3\times2\times1}{4\times3\times2\times1\times2\times1}$$
$$= 7 \times 3 \times 5$$
$$= 105$$

25) **C** The next time all lights will flick together will be a multiple of 12, 60, 45, and 56 seconds. Prime factor 12, 60, 45, and 56 to find their least common multiple, as shown to the right. Therefore, the lights will flick together again in 2520 seconds.

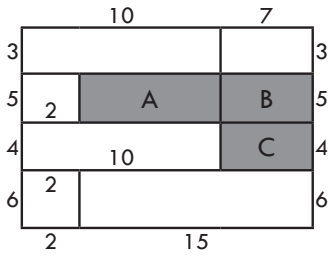
$12 = 2^2 \times 3$
$60 = 2^2 \times 3 \times 5$
$45 = 3^2 \times 5$
$56 = 2^3 \times 7$
$\text{LCM} = 2^3 \times 3^2 \times 5 \times 7$
$= 2520$ seconds

26) **D** Five minutes is $5 \times 60 = 300$ seconds. In 300 seconds, $300 \times 300 = 90\,000$ cm^3 of water is drained from a tank that was half full. A full tank has a volume of $90\,000 \times 2 = 180\,000$ cm^3.

27) **B** The lowest sum Penelope can roll is $1 + 1 + 1 + 1 = 4$ and the highest sum is $6 + 6 + 6 + 6 = 24$. Penelope can roll all possible sums from 4 to 24. That is $24 - 3 = 21$ different possible sums.

28) **A** Each interval consisting of opened doors and travel time takes $10 + 40 = 50$ seconds. To arrive at the 15th stop, there must be 14 intervals of 50 seconds, or $14 \times 50 = 700$ seconds. The doors will remain open 700 to 710 seconds after 10 a.m. Since 700 seconds is $700 \div 60 = 11$ minutes and 40 seconds, the doors are open from 10:11:40 a.m. to 10:11:50 a.m. at the 15th stop.

29) **D** In the top two rectangles, the GCF of 30 and 21 is 3, so they share a height of 3 cm. Their respective widths are $30 \div 3 = 10$ cm and $21 \div 3 = 7$ cm. The rectangle with area of 40 cm^2 has the same width as the rectangle with area 30 cm^2 so it has a height of $40 \div 10 = 4$ cm. The rectangles of area 10 cm^2 and 12 cm^2 must have the same width that is a factor of 10 and 12, so their width is 1 cm or 2 cm. Since the rectangle of area 12 cm^2 must have a height that is a factor of 90, the width of these rectangles must be 2 cm and the height of the rectangle with area 10 cm^2 is $10 \div 2 = 5$ cm. Rectangle A has a width of $10 - 2 = 8$ and a height of 5, therefore an area of $8 \times 5 = 40$ cm^2. Rectangle B has a width of 7 and a height of 5, therefore an area of $7 \times 5 = 35$ cm^2. Rectangle C has a width of 7 and a height of 4, therefore an area of $7 \times 4 = 28$ cm^2. The total area of shaded rectangles A, B, and C is $40 + 35 + 28 = 103$ cm^3.



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30) **B** Make a chart to help you count, sorting by the number of times the digit 1 appears in the ones, tens, and hundreds column. From 1 to 99, a 1 appears in the ones column in the numbers 1, 11, 21, 31, 41, 51, 61, 71, 81, and 91 for a total of 10 times. A 1 appears in the tens column in the numbers 10 to 19 for a total of 10 times. From 100 to 199, a 1 appears in the ones and tens column again 10 times each. It also appears in the hundreds column for each of 100 to 199, thus 100 times. So, the digit 1 appears 10 + 10 + 10 + 10 + 100 = 140 times from 1 to 199. The 141st number is the next appearance of a 1, which is in the number 201.

	Ones	Tens	Hundreds
1 to 99	10	10	
100 to 199	10	10	100

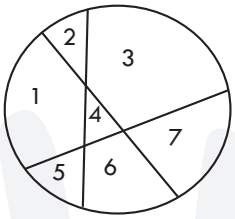
31) **B** Look for a pattern in the sum of the two leftmost digits. If the leftmost digits are two 3s, the sum is 6. Next, there is 6 + 3 = 9, followed by 9 + 3 = 12 which is not a single digit so it is added again to get 1 + 2 = 3. So, the pattern forms a repeating block of three terms: 3, 6, 9. Since 100 ÷ 3 = 33 remainder 1, the pattern will repeat 33 times and the remaining digit will be the first term in the block, which is 3.

32) **B** If the blacksmith takes 2 chains and cuts all the links in them, then he will have 12 opened links to use. Since there are 15 – 2 = 13 chains left over, they can be linked with the 12 opened links.

33) **D** Review the given options to find one that is certainly true.
Option A: False. If Augustus watches a movie, his flowers will not necessarily grow.
Option B: False. If Augustus does not watch a movie, his flowers will not necessarily grow.
Option C: False. If Augustus does not watch a movie, he will not necessarily water his flowers.
Option D: True. If Augustus' flowers grew, he did not watch a movie.

34) **C** Draw the chords in a circle and look for a pattern:

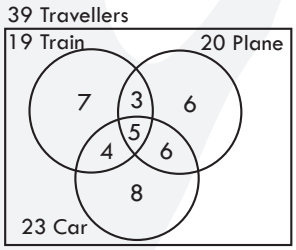
# Chords	Regions	Pattern
0	1	1
1	2	1 + 1 = 2
2	4	1 + 1 + 2 = 4
3	7	1 + 1 + 2 + 3 = 7
15	121	1 + 1 + 2 + ... + 15 = 1 + (15 × 16) ÷ 2 = 121



Therefore, there are 121 regions.

35) **B** The speed when traveling up Woby River is 24 – 8 = 16 km/h. It takes Ivy 32 ÷ 16 = 2 hours to travel up the river. The speed when traveling down Woby River is 24 + 8 = 32 km/h. It takes Ivy 32 ÷ 32 = 1 hour to travel down the river. The entire trip of 32 + 32 = 64 km takes 2 + 1 = 3 hours. Ivy's average speed is therefore $64 \div 3 = 21\frac{1}{3}$ km/h.

36) **D** There are 19 + 20 + 23 = 62 travellers who took at least one mode of transportation, but those who have taken two modes of transportation have been counted twice. So, there are 62 – 9 – 11 – 8 = 34 travellers who took one or two modes of transportation. Therefore, the remaining 39 – 34 = 5 travellers took all three modes of transportation. Use a Venn diagram to check your work.



37) **B** The 4-digit numbers are from 1001 to 9999. For a 4-digit number to be a palindrome, the first two digits should mirror the last two digits. The first two digits can be any number from 10 to 99. There are 99 – 9 = 90 numbers from 10 to 99 so there are 90 4-digit palindromes. Every third number will be divisible by 3, so 90 ÷ 3 = 30 palindromes are divisible by 3.

38) **A** The sum J + O + Y produces a different units digit each time, so there must be a different carry over each time. This can only happen when J + O + Y = 19, so R = 9. There is a carry over of 1 into the tens column, 19 + 1 = 20 and A = 0. There is a carry over of 2 into the hundreds column, 19 + 2 = 21. Hence, S = 2 and T = 1. So S + T + A + R = 2 + 1 + 0 + 9, which is 12.

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39) **A** Since the two numbers spoken by the aliens are consecutive, one is even and the other is odd. Suppose the numbers are 6 and 7. This means the number on the ship cannot be divisible by 6 or 7, but also not by 12, 14, 18, 21, 24, 28, or 30, which is not possible because Alien #11 claims the number is divisible by 12, Alien #13 claims the number is divisible by 14, and so on. Thus, each of the two numbers cannot have multiples that are less than 31 because it will also make other alien's statement incorrect. So, the numbers must be greater than $31 \div 2 = 15.5$. Consider the two numbers to be 29 and 30 meaning the number on the ship cannot be divisible by 29 and 30. This is not possible because $30 = 2 \times 3 \times 5$ and the number written on the ship is divisible by 2, 3, and 5. For instance, if a number is divisible by all of 2, 3, and 5, the number must be divisible by 30. However, since we assumed that the number on the ship is not divisible by 29 and 30, it is not possible. Another example, assume the number on the ship is not divisible by 17 and 18. It cannot be true because the number on the ship is already divisible by 2 and 9 as confirmed by other aliens so the number on the ship must also be divisible by 18 which is a contradiction. So, the odd number should be a prime number greater than 15, and the even number should be greater than 15 and have only one prime factor. Therefore, the numbers are 16 and 17 and Alien #15 and #16 were incorrect. The digit sums are $1 + 5 = 6$ and $1 + 6 = 7$, and 6 is option A.

40) **D** The pattern is to use the three outer numbers to produce the value inside the triangle. Amongst the three numbers on the outside of the triangle: one number is a prime number and one number is a multiple of five. Divide the multiple of 5 by 5 then add it to the prime number and subtract the last number outside the triangle. For example, in the first triangle, $25 \div 5 + 43 - 27 = 21$. The number in the last triangle is: $15 \div 5 + 71 - 21 = 53$.

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