2023 Grade Five Spirit of Math Contest Solutions

1) **B** Follow the order of operations and calculate: $6 \times 3 - 28 + 5 = 18 - 28 + 5$
$= -5$

2) **C** The pattern is to subtract 5 from the previous term. The next term is $22 - 5 = 17$.

3) **B** There are 63 brushes to distribute 3 to each student, so each student gets $63 \div 3 = 21$ brushes.

4) **B** From the options given, 64 is divisible by 2, 4 and 8 since $64 \div 2 = 32$, $64 \div 4 = 16$ and $64 \div 8 = 8$ respectively. Using the divisibility rule for 3, the sum of the digits $6 + 4 = 10$ is not divisible by 3 so 64 is not divisible by 3.

5) **B** The prime numbers between 1 and 20 are 2, 3, 5, 7, 11, 13, 17 and 19. Therefore, there are 8 prime numbers between 1 and 20.

6) **D** The whole pizza is cut into 8 equal sized pieces. If Jacob ate 1 piece of 8 pieces, there are $8 - 1 = 7$ pieces left out of 8 pieces. Therefore, $\frac{7}{8}$ of pizza is left.

7) **D** The largest triangle is made up of 16 smaller equal-sized triangles. Since one of the smaller triangle's area is 2 cm^2, the area of the largest triangle is $16 \times 2 = 32$ cm^2.

8) **C** Checking each place digits after the decimal point, the order of the numbers from least to greatest is 3.333, 3.335, 3.412 and 3.444. Therefore, the second largest number is 3.412.

9) **D** Since I have twice as many pencils as erasers, I have $30 \times 2 = 60$ pencils. I have three times as many crayons as pencils, so I have $60 \times 3 = 180$ crayons.

10) **C** Adding the top row gives $4 + 9 + 2 = 15$. Looking at the diagonal from top left corner to the bottom right corner, $4 + * + 6 = 15$. Therefore, the missing number "*" is 5.

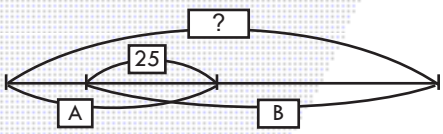
11) **C** The next time Roham will drink and eat at the same time will be at a multiple of 10 and 12. The least common multiple of 10 and 12 is 60. Therefore, Roham will drink and eat at the same time again in 60 minutes which is at 9:00 a.m.

12) **B** If each puppy eats 100 g of dog food, 14 puppies will eat $14 \times 100 = 1400$ grams of dog food. Since a bag of dog food contains 1000 g, you need 2 bags of dog food in order to feed all 14 puppies.

13) **B** If the weight of the box only is 10 g, the weight of the chocolates only is $70 - 10 = 60$ g. Since there are 12 identical chocolates, each chocolate weighs $60 \div 12 = 5$ grams.

14) **A** Looking at each fractions, half of the denominator 9 of the first fraction is 4.5 so $\frac{8}{9}$ is more than a half. Half of 15 is 7.5 so $\frac{14}{15}$ is also more than a half. Half of 7 is 3.5 so $\frac{4}{7}$ is more than a half. Half of 10 is 5 so $\frac{3}{10}$ is less than one half. Half of 180 is 90 so $\frac{37}{180}$ is less than one half. Therefore, two fractions are less than one half.

15) **B** Adding the lengths on the bottom gives $32 + 55 = 87$. However, the segment with length of 25 is included both in the segments A and B so it has been added twice. Therefore, the length of the whole line segment is $87 - 25 = 62$.



16) **C** There are 5 choices for the first letter, and for each of these choices, there are 4 choices for the second letter, and for each of these choices, there are 3 choices and so on. Altogether, there are $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ arrangements. However, there are two B's in the word TABBY. Arranging these two B's will give the same word. For example, $TAYB_1B_2$ and $TAYB_2B_1$ are the same words. Therefore, the total number of arrangements needs to be divided by 2! giving $120 \div 2 = 60$ different ways to arrange the letters in the word TABBY.

17) **D** Looking at the first diagram, the faces with 54 and 23 share the same edge as the face with 10 so these faces cannot be opposite to the face with 10. Observing the second diagram, the face with 54 is still on the top so this cube has been rotated from the first diagram. Since the face with 10 is adjacent to the face with 23 and the face with 23 is adjacent to the face with 67, the face with 67 must be opposite to the face with 10 on it.

18) **C** The distance between the first person and the last person in line was approximately 1 km which is equivalent to 1000 metres. Since there are 2 metres between each person, there are $1000 \div 2 = 500$ spaces between the first person and the last peron in line. If there are 500 spaces or intervals, there are $500 + 1 = 501$ people lined up for the ice cream.

19) **D** If the average of a set of 8 numbers is 15, the sum of the 8 numbers is $8 \times 15 = 120$. When the number 15 is removed from the set, the sum of a set of 7 numbers is $120 - 15 = 105$. Therefore, the average of the remaining 7 numbers is $105 \div 7 = 15$.

20) **A** The distance of 20 m is equivalent to $20 \text{ m} \div 1000 = 0.02$ km (since 1000 m = 1 km) so the scooter travels 0.02 km in 2 seconds.
There are 3600 seconds in 1 hour so the rate can be converted: $\dfrac{0.02 \text{ km} \times 1800}{2 \text{ seconds} \times 1800} = \dfrac{36 \text{ km}}{3600 \text{ seconds}}$
$= 36 \text{ km/h}$
Alternative solution using equivalent ratios (fractions):

$$\frac{0.02 \text{ km}}{2 \text{ seconds}} = \frac{x \text{ km}}{3600 \text{ seconds}}$$
$$x = 0.02 \times 3600 \div 2$$
$$= 36 \text{ km/h}$$

21) **B** The first multiple of 13 after 100 is 104 ($13 \times 8 = 104$) and the last multiple of 13 before 400 is 390 ($13 \times 30 = 390$). There are 30 multiples of 13 until 400 and the first 7 of these numbers are below 100. Therefore, there are $30 - 7 = 23$ multiples of 13 between 100 and 400.

22) **C** The first Olympics was held in 1896, then the second Olympics was held in $1896 + 4 = 1900$ and the third Olympics was held in $1896 + 4 + 4 = 1904$ and so on. For the 4th Olympics, you would need to add three 4's to 1896 so for each nth Olympics, you need need to add n – 1 fours to the year 1896. Therefore, the 20th Olympics would be held in $1896 + (19 \times 4) = 1972$.

23) **C** To get the largest possible difference between the two numbers, you need to subtract the smallest possible number from the largest possible number. The largest number you can round to the nearest hundred to get 3700 is 3749 because you want the tens digit to be less than 5. Similarly, the smallest number you can round to the nearest hundred to get 2800 is 2750 because you want the tens digit to be greater than 4. Therefore, the difference of these two numbers is $3749 - 2750 = 999$.

24) **B** Since there are 6 prime numbers from 1 to 13 (2, 3, 5, 7, 11, 13), there are $6 \times 2 = 12$ prime number cards that are black (since there are two black cards for each number). There are 52 cards to choose from so the probability that the card chosen is a black prime number is $\dfrac{12}{52} = \dfrac{3}{13}$

25) **D** The minute hand rotates 360° in an hour which is a full circle. The minute hand of a clock moves $360° \div 60 \text{ min} = 6°$ every minute. For 3 hours, the minute hand moved $360° \times 3 = 1080°$ and for 40 minutes, it moved $6° \times 40 = 240°$. Therefore, the minute hand moved $1080° + 240° = 1320°$.

26) **D** Rhiya paid \$7 for 5% sales tax so \$7 is 5% of the toy's regular price. The regular price of the toy is $7 \div 0.05 = 140$ dollars. Since there is a discount of 20% off, the discount price is 80% of the regular price which is $140 \times 0.8 = 112$ dollars. There is also 5% sales tax on top of the discount price so the final price after the discount and tax is $112 \times 1.05 = 117.60$ dollars.

27) **B** The perimeter is the distance around the entire shape. The bottom length of the entire figure is $1 + 2 + 3 + 4 + ... + 20 = 210$ cm and the length of the top part of the entire figure is also 210 cm. The length of the side on the right is 20 cm which is the length the 20th square. The length of the side on the left is also 20 cm. Therefore, the perimeter of the entire figure is $210 + 210 + 20 + 20 = 460$ cm.

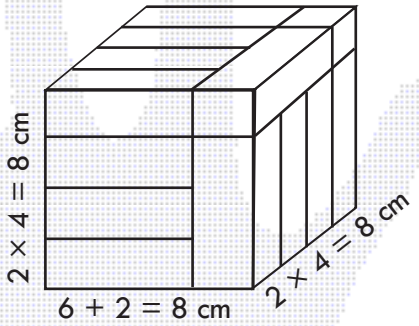
28) **A** Let's assume that Igor's statement is true, then Dima and Svitlana's statements are also true so this is not possible. Assume that Svitlana's statement is true, then Igor and Dima's statements are also true so this is also not possible. This means only Dima's statement is true and Igor and Svitlana's statements are false. Therefore, Igor did not eat the doughnut and Svitlana did not eat the doughnut, leaving only Dima who ate the doughnut.

29) **B** The two-digit multiples of 7 are 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91 and 98. Among these numbers, numbers that have tens digit greater than the ones digit are 21, 42, 63, 70, 84 and 98. The only numbers with a difference between the tens digit and ones digit of 1 among these numbers are 21 and 98. The sum of these two numbers is $21 + 98 = 119$.

30) **C** In this game, Dora will have 4 rabbits after one week, 16 rabbits after second week and 64 rabbits after third week and so on. The number of rabbits are all powers of 4: $4^1 = 4$, $4^2 = 16$, $4^3 = 64$ and so on where the exponent is the number of weeks. Since $256 = 4^4$, it takes 4 weeks or $4 \times 7 = 28$ days to get 256 rabbits. Dora will also have 2 squirrels after 4 days, 4 squirrels after 8 days, 8 squirrels after 12 days and so on. The number of squirrels are all powers of 2: $2^1 = 2$, $2^2 = 4$, $2^3 = 8$ and so on where number of days is the exponent multiplied by 4 (since it doubles every four days). Since $64 = 2^6$, it will take $6 \times 4 = 24$ days to get 64 squirrels. However, since Dora needs more than 64 squirrels, she needs 4 more days after 24 days. Therefore, Dora needs to play at least 28 days.

31) **C** From 1 to 99, the digits 3 and 7 are printed 20 times each: 10 times in the units digit and 10 times in the tens digit. From 100 to 199, this is another 20 times printed for each digit. From 200 to 237, the digit 3 is printed 14 times: 10 times in the tens digit and 4 times in the units digit. The digit 7 is printed 4 times in the units digit from 200 to 237. Therefore, the digits 3 and 7 are printed $40 + 40 + 14 + 4 = 98$ times.

32) **B** To use the least number of blocks, Mary needs to use as many 6 by 2 by 2 blocks as possible. As seen on the diagram on the right, she can stack 16 of these blocks on the left side to make a 6 by 8 by 8 figure. For the remaining 2 cm on the right, Mary can rotate the 6 by 2 by 2 blocks and use 4 of these blocks vertically. On top of these blocks, she can fit another 6 by 2 by 2 blocks and lastly, fit one 2 by 2 by 2 block at the very back. Therefore, Mary needs $16 + 6 = 22$ blocks.



33) **C** Using 2, 3, 6 and 10, the largest possible number can be obtained by multiplying the two largest numbers and then dividing by the smallest number: $3 + 6 \times 10 \div 2 = 33$. Similarly, the smallest number can be obtained by dividing the largest possible number to get the smallest quotient: $2 + 10 \times 3 \div 6 = 7$. The difference between these two numbers is $33 - 7 = 26$.

34) **C** The first few factorials are: $1! = 1$, $2! = 2$, $3! = 6$, $4! = 24$, $5! = 120$ and so on. After 5!, the units' digits of the factorials will always equal to zero because you keep multiplying the next digit to a number ending in 0. Since you are looking for the unit's digit of the sum, add the factorials before 5!, $1 + 2 + 6 + 24 = 33$ which gives the unit's digit of 3.

35) **D** There are 6 faces in a cube and for each face of the cube, Sarah can rotate it horizontally 4 times. Therefore, there are $6 \times 4 = 24$ ways to fit the cube in the box.

36) **B** The powers of 5 that have at least four digits start with 5^5: $5^5 = \mathbf{3125}$, $5^6 = 1\mathbf{5625}$, $5^7 = 7\mathbf{8125}$, $5^8 = 39\mathbf{0625}$ and so on. Since you only need to look at the last four digits, multiplying $5^8 = 390625$ by 5 will give 5^9 ending in 3125. There is a pattern of four digit numbers that keep repeating with each power of 5: 3125, 5625, 8125, 0625. Dividing the exponent $356 \div 4 = 89$, you find that the pattern repeats 89 times and therefore will end with 0625.

37) **C** Work backwards. Half an hour before the two trains meet, each train would have to travel for 30 minutes until the two trains meet. One train travels at 168 km/h so this train will travel $168 \div 2 = 84$ km in half an hour and the other train travels at 142 km/h so this train will travel $142 \div 2 = 71$ km in half an hour. Therefore, the two trains are $84 + 71 = 155$ km apart.

38) **C** Let T represent the total number of candies then the amount of candies each person has can be represented as following:

Amir = $\frac{1}{4}$ of T – 6 Betty = $\frac{1}{6}$ of T + 3 Carl = Amir + 3

Carl took three more than Amir so the number of candies Carl has can be expressed by:

Carl = $\frac{1}{4}$ of T – 6 + 3

= $\frac{1}{4}$ of T – 3

Adding the amount of candies for all three siblings: $(\frac{1}{4} + \frac{1}{6} + \frac{1}{4})$ of T – 6 = $\frac{2}{3}$ of T – 6 which is 6 less than two-thirds of total number of candies.

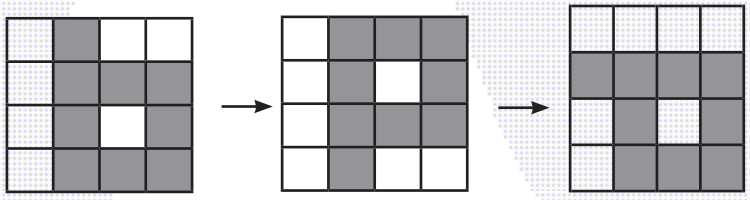
So the number of candies left is one-third of total number of candies plus 6:

20 = $\frac{1}{3}$ of T + 6

(20 – 6) × 3 = Total

Therefore, they started with 42 candies.

39) **A** If Jasmine flipped the picture vertically 9 times, this would be the same as flipping the picture vertically one time because the picture comes back to the original position when flipped twice. So after flipping the picture vertically 9 times, the picture will look like diagram 1 below. Then, Jasmine rotated the picture 270 degrees counter-clockwise 13 times. Rotating 270° counter-clockwise is the same as rotating 90° clockwise because a full rotation is 360°. When a picture is rotated 90° clockwise four times, the picture will come back to its original position. If the picture is rotated 90° clockwise 13 times, this would be the same as rotating the picture 90° clockwise one time and the picture will look like diagram 2 below after rotated.



Start Diagram 1 Diagram 2

40) **C** The pattern is to multiply two largest numbers on the corner of the triangle then subtract by the square of the third number on the triangle. As an example, the middle number on the first triangle is $5 \times 7 - 3^2 = 26$ and the second triangle: $6 \times 4 - 2^2 = 20$. Therefore, the missing number on the last triangle is $7 \times 6 - 4^2 = 26$.