2023 Grade Six Spirit of Math Contest Solutions

1) **A** Follow the order of operations and calculate: $-4 \times 7 + 25 + 7 = -28 + 25 + 7$
$= 4$

2) **C** All numbers in the set are prime numbers, except 25.

3) **B** The number of sides of a hexagon is 6 and the number of sides of an octagon is 8. Therefore, the sum of the number of sides is $6 + 8 = 14$.

4) **D** If the perimeter of a square is 48 cm, the length of one side of the square is $48 \div 4 = 12$ cm. To find the area of the square, multiply the length by width which is $12 \times 12 = 144$ cm^2.

5) **B** To find 10% of 1000, multiply 1000 by 0.1 (10%): $1000 \times 0.1 = 100$. Therefore, only 100 people will be allowed in the mall.

6) **D** The ratio of purple pencils to yellow pencils is 3:13 and since there are 9 purple pencils, the number of purple pencils has been tripled. So there are $13 \times 3 = 39$ yellow pencils.

7) **B** Since only the units digit of the product is needed, multiply the units digit of 2348 and 3987 which is $8 \times 7 = 56$. The units digit of 56 is 6 so the product will have a units digit of 6.

8) **C** If Yui is 12 years old, then Mei is $12 - 5 = 7$ years old. If Mei is 7 years old, then Bao is $6 + 7 = 13$ years old.

9) **C** If you travel 65 km in 50 minutes, then in 10 minutes, you travel $65 \div 5 = 13$ km. There are 60 minutes in an hour so you travel $13 \times 6 = 78$ km/h.

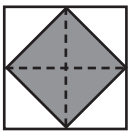
10) **B** The area of a square is calculated by length multiplied by width where the length and width are the same. If the area of a square is 36 cm^2, then the length of the square is 6 cm. If a rectangle has the same width of 6 cm and a length twice as long, $6 \times 2 = 12$ cm, the perimeter of the rectangle is $(6 + 12) \times 2 = 36$ cm.

11) **C** The worst case scenario is when you choose all roses except the blue roses. So you will choose $11 + 7 = 18$ roses that are not blue roses. Then, you must choose 2 more blue roses to be certain that you have chosen 2 blue roses. Therefore, you must choose at least $18 + 2 = 20$ roses.

12) **D** There are 6 choices for the first letter, and for each of these choices there are 5 choices for the second letter, 4 choices for the third letter, 3 choices for the fourth letter, 2 choices for the fifth letter and 1 choice for the sixth letter. Altogether, therer are $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ different ways to arrange the letters in the word GENIUS.

13) **B** The lemon orchard has $60 \times 80 = 4800$ lemons in total. Since each lemon produces 50 mL of lemon juice, there will be a total of $4800 \times 50 = 240\,000$ mL of lemon juice. In litres, there will be $240\,000 \text{ mL} \div 1000 = 240$ L of lemon juice that the orchard can produce.

14) **B** The tax will add 10% to the total cost of the bouquet. Since 10% of \$50 is $50 \times 0.1 = \$5$, the final price of the bouquet is $\$50 + \$5 = \$55$.

15) **C**  When the larger square is cut into four equal smaller squares as shown on the left, it is shown that the shaded region in each smaller square is half the area of the each smaller squares. Therefore, the area of the larger square will be double of the shaded region, $221 \times 2 = 442$ cm^2.

16) **B** The total length of the line segment is $54 - 12 = 42$. The distance between two consecutive points is $42 \div 6 = 7$, since there are 6 intervals on the line segment. There are 4 intervals from the starting point to point A so point A will represent $12 + (7 \times 4) = 40$.

17) **C** Simplify by converting the denominator to an improper fraction: $2+\frac{3}{1\frac{2}{3}}=2+\frac{3}{\frac{5}{3}}$

A fraction line implies division, so $2+3\div\frac{5}{3}=2+3\times\frac{3}{5}$

$$=2+\frac{9}{5}$$

$$=3\frac{4}{5}$$

18) **C** There are 3 choices for a top, 4 choices for pants and 5 choices of pairs of shoes. Altogether, there are $3 \times 4 \times 5 = 60$ different ways to arrange Aashi's outfits.

19) **B** The probability of getting a letter "c" from "a" to "f" is $\frac{1}{6}$. The probability of getting a number greater than "4" from a "1" to "6" is $\frac{1}{3}$. Therefore, the probability of getting a letter "c" and a number greater than "4" is $\frac{1}{18}$.

20) **B** Insert a = 6 and b = 3 to get: $6^2 - 3 \times 6 + 3 \times 2 = 36 - 18 + 6$

$$= 24$$

21) **C** Between each pair of adjacent streetlights, there is a space of 8 metres. Since there are 31 streetlights in total, there are 31 spaces between the streetlights. Therefore, the circumference of this park is $31 \times 8 = 248$ m.

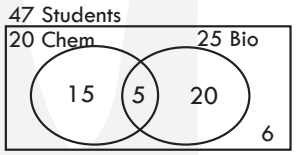
22) **C** The surface area of the walls except the floor is :

$(8 \times 3) + (8 \times 3) + (6 \times 3) + (6 \times 3) + (8 \times 6) = 24 + 24 + 18 + 18 + 48$

$$= 132 \text{ m}^2$$

The area of the ceiling is $8 \times 6 = 48$ and since only the ceiling is painted yellow, the fraction of the area painted in yellow is $\frac{48}{132}=\frac{4}{11}$.

23) **D** Use a Venn diagram to solve this problem. If 5 students went to both labs, then $20 - 5 = 15$ students went to only chemistry lab and $25 - 5 = 20$ students went to only biology lab. Therefore, $47 - 15 - 20 - 5 = 7$ students went to none of these labs.



24) **C** The diagram in the question is 5 by 6 rectangle. Find the number of squares starting with the smallest square. There are $5 \times 6 = 30$ one-by-one squares, $4 \times 5 = 20$ two-by-two squares, $3 \times 4 = 12$ three-by-three squares, $2 \times 3 = 6$ four-by-four squares and $1 \times 2 = 2$ five-by-five squares. Therefore, there are $30 + 20 + 12 + 6 + 2 = 70$ squares of different sizes.

25) **D** Assuming A is 100, 50% of A = 100 is 50. Then, 50 is 25% of B so $B = 50 \div 0.25 = 200$. Therefore, B is $(200 \div 100) \times 100 = 200\%$ of A.

26) **C** This sequence $7 + 13 + 19 + \ldots + 607$ is an arithmetic sequence. The average of these numbers in the sequence is $(607 + 7) \div 2 = 307$ and the number of terms in the sequence is $(607 - 1) \div 6 = 101$. The sum of these numbers in the sequence can be found by multiplying the average of the numbers and the number of terms (numbers). Therefore, the sum is $307 \times 101 = 31007$.

27) **B** If the number has a quotient of 27 when divided by 19, then the dividend is $19 \times 27 = 513$. Since the three-digit number must have a remainder when divided by 19 with a quotient of 27, the three-digit number must be greater than 513 and less than $19 \times 28 = 532$. The three-digit numbers that can be made using 5, 0, 9, 2, and 3 that are between 513 and 532 are 520, 523, 529 and 530. Therefore, there are 4 numbers Birte can make.

28) **B** If Jamie sold $\frac{1}{5}$ of the chocolates on Monday and had $\frac{1}{4}$ of chocolates left at the end of Tuesday, he sold $1-\frac{1}{5}-\frac{1}{4}=\frac{11}{20}$ of chocolates on Tuesday. So, 220 chocolates is $\frac{11}{20}$ of the total number of chocolates. The total number of chocolates he made is: $\frac{220}{Total}=\frac{11}{20}$

$$Total = 220 \times 20 \div 11$$
$$= 400$$

At the end of Tuesday, Jamie has 400 ÷ 4 =100 chocolates left to sell. If he plans to sell one half of the remaining chocolates, Jamie needs to sell 100 ÷ 2 = 50 chocolates on Wednesday.

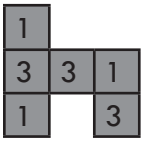
29) **A** If 8 teachers can mark 8 tests in 8 minutes, then 100 teachers can mark 100 tests in 8 minutes. The rate of each teacher marking each test does not change. If each teacher marks double the number of tests, it will take twice as long. So, 100 teachers can mark 200 tests in 16 minutes.

30) **C** When 2 and 5 are multiplied, it gives 10 and produces a zero at the end of a number. The number of pairs of 2 and 5 will determine how many zeros the number will end in. In $2^{444} \times 5^{446}$, there are 444 pairs of 2 and 5 multiplied: $5^2 \times 2^{444} \times 5^{444}$ and there are 444 zeros at the end of the product. There is also $5^2 = 25$ so there are 444 + 2 = 446 digits in the number.

31) **B** Claire takes 3 hours to shovel one driveway and her dad takes 1 hour. In 3 hours, Claire can shovel 1 driveway and her dad can shovel 3 driveways so they can shovel 3 + 1 = 4 driveways together. To shovel one driveway together, it will take 3 ÷ 4 = 0.75 hours = 45 minutes.

32) **B** Breaking even means Pam does not make any profit or lose any money. If it costs Pam $2 to make each cupcake and she sells them for $5 each, then she makes a profit of 5 – 2 = $3 per cupcake. Pam spent $435 on her tools so she needs to sell 435 ÷ 3 = 145 cupcakes to break even.

33) **B** When this figure is seen from the front, the height of each column is three blocks. So the middle column must have 3 blocks going up and left and right columns must have 3 blocks going up in one of the row. When seen from the right side, the rightmost column must have only one block going up. After stacking the highest blocks in each column, the rest must have one block only since the question is asking for the least number of blocks. Therefore, 1 + 3 + 3 + 1 + 1 + 3 = 12 blocks are needed. The number of blocks stacked can be represented as on the diagram on the right.



Top

34) **C** The average speed is calculated by the total distance divided by the total time. In 5 hours, the bicycle will travel 10 × 5 = 50 km. The bus started 180 km behind the bicycle so by the time the bus catches up to the bicycle, the bus has travelled 50 + 180 = 230 km. The total time it took for the bus to catch up is 5 hours. Therefore, the average speed is 230 km ÷ 5 hours = 46 km/h.

35) **A** There are 100 cm in 1 metre. The volume is calculated by length × width × height so the volume of a cube with 100 cm length is 100 × 100 × 100 = 1 000 000 cm^3 and the volume of a cube with 1 metre length is 1 m^3. Therefore, the volume of a rectangular prism in m^3 is 72 000 ÷ 1 000 000 = 0.072 m^3.

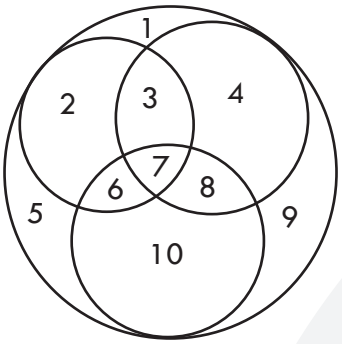
36) **D** The sum of all numbers 11, 12, 13, 14, 15, 16, 17, 18 and 19 is 135. Dividing 135 ÷ 3 = 45 gives the sum of three numbers when added vertically, horizontally and diagonally. Since these numbers are consecutive numbers from 11 to 19, place the median which is the middle number 15 in the middle of the magic square. Then, the largest number 19 can to be placed in the same column as 11 where B is. The second largest number 18 can placed in the same diagonal line as 12 where A is. Therefore, A + B is 19 + 18 = 37.

16	11	18
17	15	13
12	19	14

37) **D** Determine which statement is true to find out the colour of the marker each sibling is using. In Case 1, both Kai and Max are using a pink marker, which is not possible. In Case 2, Max is not using a pink marker, Kai is not using a pink marker, and Roy is using a blue marker. If both Max and Kai are not using a pink marker, then they must be using a blue or green marker. But Roy is using a blue marker, so this is not possible. Thus the answer must be Case 3 where Roy is not using a blue marker, Kai is not using a pink marker, and Max is using a pink marker. Therefore, Roy is using a green marker and Kai is using a blue marker.

	Case 1	Case 2	Case 3
Kai uses a pink marker	T	F	F
Max is not using a pink marker	F	T	F
Roy is not using a blue marker	F	F	T

38) **C** Look for a pattern between the number of smaller circles and the greatest number of regions the big circle gets divided into. Try with 3 smaller circles to see at most how many regions you can make, there are at most 10 regions the big circle can be divided into as seen from the below diagram. Similarly, there are at most 17 regions you can make with 4 smaller circles inside the big circle. The pattern to get the most number of regions is to square the number of small circles and add 1.



# of small circles	# of regions
1	$2 = 1 \times 1 + 1$
2	$5 = 2 \times 2 + 1$
3	$10 = 3 \times 3 + 1$
4	$17 = 4 \times 4 + 1$

Therefore, the greatest number of regions you can get with 9 circles would be $9 \times 9 + 1 = 82$.

39) **C** When you divide any number by 10, the remainder is the units digit of the dividend. So, find the units digit of each power and add them up. Working out the first few powers of 2 you have: $2^1 = \mathbf{2}$, $2^2 = \mathbf{4}$, $2^3 = \mathbf{8}$, $2^4 = 1\mathbf{6}$, $2^5 = 3\mathbf{2}$,... The units digits are, in order, 2, 4, 8, 6, 2, 4, ... There is a pattern of four digits: 2, 4, 8, 6 that keep repeating. Dividing the exponent 102 by 4 you find that the pattern repeats 25 times, and then has two extra digits ($102 \div 4 = 25$ remainder 2). So the units digit in 2^{102} must be **4**, the second digit in the pattern. Similarly, the units digits of powers of 3 has a pattern of four digits: 3, 9, 7, 1 that keep repeating. For 3^{103}, dividing the exponent of 103 by 4 gives a quotient of 25 with a remainder of 3 so the unit's digit in 3^{103} must be **7**, the third digit in the pattern. Powers of 4 have units digits ending in 4, 6, 4, 6, ... where the powers with even exponents all end in **6**. Lastly, the unit's digit of 7^{107} is **3** following the same logic as above. Therefore, the sum of the remainders when you divide each number 2^{102}, 3^{103}, 4^{104}, and 7^{107} by 10 is $4 + 7 + 6 + 3 = 20$.

40) **A** For each square, you need to use the numbers on the top row and on the bottom left to get a number on the bottom right. The pattern to get the number on the bottom right is $(\text{largest number})^2 - (\text{second largest number})^2 + \text{smallest number}$. For example, for the first square, $8^2 - 5^2 + 3 = 42$. Therefore, the missing number on the last square is $8^2 - 7^2 + 6 = 21$.