2024 Grade Six Spirit of Math Contest Solutions

1) **A** Follow the order of operations and calculate: $-7 \times (3 + 4) - 14 = -7 \times 7 - 14$
$= -49 - 14$
$= -63$

2) **D** Since $51 \div 3 = 17$, 51 is divisible by 1, 3, 17, and itself. Thus, 51 it is not a prime number. The numbers 17, 23, and 47 are prime.

3) **C** The number of vertices of a pentagon is five and the number of vertices of a hexagon is six. Therefore, the sum of the number of vertices is $5 + 6 = 11$.

4) **A** The pattern is to multiply the previous term by five. Therefore, the next term is $5 \times 500 = 2500$.

5) **C** Two minutes is equal to $2 \times 60 = 120$ seconds. If Eli can drink one cup of milk in 10 seconds, she can drink $120 \div 10 = 12$ cups of milk in two minutes.

6) **C** Since $20\% = 0.2$, the amount of juice in your container is $5 \times 0.2 = 1$L.

7) **B** A full circle is 360°. There are 6 equal slices, so one slice is $360 \div 6 = 60°$.

8) **B** There are $2 + 3 = 5$ students who are not 11 years old. Since the total number of students in the class is 20, the probability of choosing a student who is not 11years old is $\frac{5}{20} = \frac{1}{4}$.

9) **D** Rewrite the expression as $2 \times 3 \times 5 \times 5 \times 5 \times 9 = 2 \times 3 \times 5 \times 5 \times 5 \times 3 \times 3$ and combine the same prime numbers to get $2 \times 3^3 \times 5^3$.

10) **D** The worst case scenario is if all apples are chosen except the green apples. There is a total of $10 + 6 = 16$ apples that are not green. Since the remaining apples in the basket are all green, the following three chosen apples will all be green. Therefore, the least number of apples you must choose to be certain to have three green apples is $16 + 3 = 19$.

11) **B** The factor set of 120, listed in pairs is: {1, 120, 2, 60, 3, 40, 4, 30, 5, 24, 6, 20, 8, 15, 10, 12}. The two numbers from this set that have a difference of 19 and a product of 120 are 5 and 24. The sum of these two number is $5 + 24 = 29$.

12) **C** A) "Does Gregory walk at a constant rate?" is not relevant to solve this problem because, in this instance, the rate of speed does not affect the distance travelled.
B) "How many light posts are on Gregory's street? is provided in the question.
C) "How many spaces are there from the first to the last post?" is the first logical question you should ask to help solve the problem.
D) "What is the distance between two neighbouring light posts?" is provided in the question.

13) **D** Since each row, column, and diagonal has a sum of 50, the missing number in the last column is $50 - 16 - 10 - 5 = 19$. Then, the missing number in the top left cell is $50 - 19 - 13 - 6 = 12$. Therefore, from the first column, the value of m is $50 - 12 - 7 - 14 = 17$.

14) **B** The cubic toy box has a volume of $4 \text{ cm} \times 4 \text{ cm} \times 4 \text{ cm} = 64 \text{ cm}^3$ and the rectangular blocks each have a volume of $1 \text{ cm} \times 2 \text{ cm} \times 4 \text{ cm} = 8 \text{ cm}^3$. At most, the toy box can hold $64 \div 8 = 8$ small blocks.

15) **B** The sum of the number of roses from both years is 56 and the number of roses from last year is three times the number of roses from this year. Thus, four times the number of this year's roses is 56. Therefore, there are $56 \div 4 = 14$ roses in Lana's garden this year.

16) **A** If each girl has two sisters, then each family has three girls. Therefore, the total number of girls on Elm street is $3 \times 5 = 15$. Since the ratio between boys to girls is 1:5, there are five times more girls than boys on the street. Therefore, there is a total of $15 \div 5 = 3$ boys living on Elm street.

17) **C** The height of the plant is 1cm at the end of the first week, $1 \times 2 = 2$ cm at the end of the second week, $2 \times 2 = 4$ cm at the end of the third week and so on. The height of the plant doubles after each week. Therefore, its height is $4 \times 2 = 8$ cm after the fourth week, $8 \times 2 = 16$ cm after the fifth week, $16 \times 2 = 32$ cm after the sixth week, $32 \times 2 = 64$ cm after the seventh week, and $64 \times 2 = 128$ cm at the end of the eighth week.

Alternate Solution:
Make a chart and record the height of the plant after each week. Notice that the height of the plant has been doubling from week to week, so the heights of the plant can be presented as 1; $2 = 1 \times 2$; $4 = 1 \times 2^2$; $8 = 1 \times 2^3$; and so on. Also, notice that the exponent of 2 in each of the expressions that represent the plant's height is equal to the number of the corresponding week minus one.
Therefore, the height of the plant after eight weeks is $1 \times 2^7 = 128$ cm.

week	height
1	1
2	2
3	4
4	8
...	...

18) **D** There are six choices for an appetizer, five choices for a main course, and three choices for a dessert. Altogether, there are $6 \times 5 \times 3 = 90$ different ways for Kenzo to choose his dinner.

19) **B** Simplify by following the order of operations:

$$2+\frac{1-\frac{1}{4}}{1+0.75} = 2+\left(1-\frac{1}{4}\right)\div(1+0.75)$$
$$= 2+\frac{3}{4}\div 1.75$$
$$= 2+\frac{3}{4}\div\frac{7}{4}$$
$$= 2+\frac{3}{4}\times\frac{4}{7}$$
$$= 2+\frac{3}{7}, \text{or } 2\frac{3}{7}$$

20) **D** The circumference of the circle is $C = 2 \times 16\pi$ or 32π cm. Since there are eight evenly spaced points on the circle, the distance between two consecutive points is $32\pi \div 8 = 4\pi$ cm. Therefore, the distance between points A and D is $3 \times 4\pi = 12\pi$ cm.

21) **D** Every week, Olivia sells $35 \times 3 = \$105$ worth of lemonades. Her weekly profit is $0.2 \times 105 = \$21$. Therefore, Olivia's profit after three weeks is $3 \times 21 = \$63$.

22) **C** Two digit numbers which are perfect squares are: 16, 25, 36, 49, 64, and 81
Check each number to see which of the options is correct.

$(16 + 4) \times 2 = 40$,
$(25 + 5) \times 2 = 60$,
$(36 + 6) \times 2 = 84$,
$(49 + 7) \times 2 = 112$,
$(64 + 8) \times 2 = 144$,
$(81 + 9) \times 2 = 180$.

Of these six solutions, only 112 is given as an option.

23) **C** Multiply 1 by 1000 to convert kilograms to grams.

Check each option:

A) 1000 g = 2 × 200 g + 2 × 300 g (two 200 g packages and two 300 g packages)

B) 1600 g = 2 × 500 g + 2 × 300 g (two 500 g packages and two 300 g packages)

C) There is no way to combine four packages of the given sizes to buy a total of 1.9 kg of ice cream.

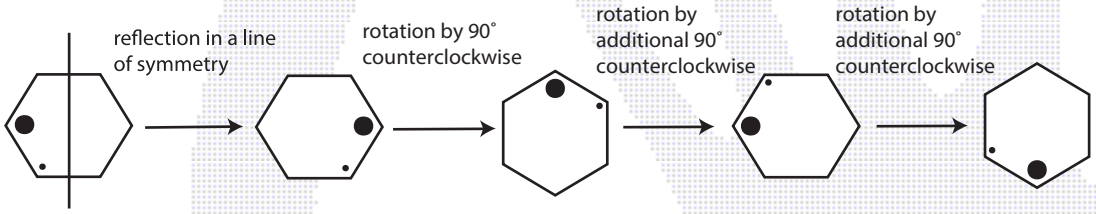
D) 2000 g = 4 × 500 g (four 500 g packages)

24) **A** Kareem's total shooting score after the first four games is 4 × 81 = 324. To achieve an average of 84%, the total score after seven games should be 7 × 84 = 588. Therefore, to achieve his goal, Kareem should score a total of 588 – 324 = 264 points in the next three games or an average of 264 ÷ 3 = 88%.

25) **A** The next palindrome after 2992 is 3003. The bus traveled a total of 3003 – 2992 = 11 km in 30 minutes. Therefore, the bus was traveling at a speed of 11 × 2 = 22 km/h.

26) **D** This series 4 + 8 + 12 + 16 + ... + 204 + 208 is an arithmetic series. The average of the numbers in the series is (4 + 208) ÷ 2 = 106 and the number of numbers in the series is 208 ÷ 4 = 52 . The sum can be found by multiplying the average and the number of numbers. Therefore, the sum is 106 × 52 = 5512. Alternatively, notice that the sum of the first and the last number (4 + 208 = 212) is equal to the sum of the second and the second last number (8 + 204 = 212) and so on. Since there are 52 numbers in the series, there are 52 ÷ 2 = 26 pairs of numbers. Therefore, the sum can be found by multiplying the sum of each pair by the number of pairs or 212 × 26 = 5512.

27) **C** Since the angle of rotation by 990° can be presented as 360° + 360° + 270°, the hexagon has rotated two full circles plus 270°. Since a figure coincides with itself as its rotates through 360°, only the rotation by 270° should be considered. The following diagram shows the transformations applied to the hexagon. The hexagon is reflected in a line of symmetry and rotated 90° counterclockwise three times for a total of 90° + 90° + 90° = 270°counterclockwise. The final shape corresponds to option C.



28) **C** Each of the nine friends gives one "high-five" to their eight other friends, which results in 9 × 8 = 72 "high-fives". However, each "high-five" has been counted twice, so there are actually 72 ÷ 2 = 36 "high-fives". As two of your friends did not attend school this morning, there are 9 – 2 = 7 remaining friends. Among the remaining friends, there are 7 × 6 ÷ 2 = 21 "high-fives". Therefore, there are 36 – 21 = 15 fewer "high-fives" among the remaining group of friends compared to those of the full group.

29) **D** From $4^1 = 4$, $4^2 = 16$, $4^3 = 64$, $4^4 = 256$, and so on, it is evident that the unit digits repeat in blocks of two digits in this order: 4 and 6. Since $122 \div 2 = 61$, the block would repeat 61 times and the last digit would be the second digit in the block, which is 6. Therefore, the last digit of 4^{122} is 6. The last digit of any power of 5 is 5 ($5^1 = 5$, $5^2 = 25$, $5^3 = 125$, and so on), so the last digit of 5^{122} is 5. The last digit of any power of 6 is 6 ($6^1 = 6$, $6^2 = 36$, $6^3 = 216$, and so on), so the last digit of 6^{122} is 6. Therefore, the sum of the last digits is $6 + 5 + 6 = 17$.

30) **A** Make a chart to help you organize the given information. Start with Samara's possible age and set her grandmother's age to 14 times that value. Then, add nine to both sets of ages. Since Samara's grandmother will be five times as old as Samara in nine years, the only possible option is if Samara will be 13 and her grandmother will be $5 \times 13 = 65$ years old. Therefore, Samara was born in 2020.

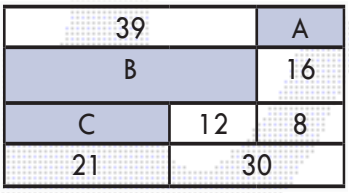
Year of Samar's Birth	2020	2021	2022	2023
Samara's age now	4	3	2	1
Grandmother age now	56	42	28	14
Samara's age in nine years	13	12	11	10
Grandmother age in nine years	65	51	37	23

31) **D** If the red and blue cubes are always placed beside each other, imagine they are "one cube". There are $4! = 4 \times 3 \times 2 = 24$ ways to arrange "four" cubes in a row. For each of these 24 ways, there are $2! = 2$ ways for the red and the blue cube to be arranged among themselves (red and blue or blue and red). Altogether, there are $24 \times 2 = 48$ ways for the cubes to be arranged such that the red and blue cubes are always beside each other.

32) **B** The operation $x \lozenge y$ is defined as $-[(x - y)^3 + y] \div (-x)$. Substitute 4 for x and 8 for y to get:

$$\begin{aligned} -[(4 - 8)^3 + 8] \div (-4) &= -[(-4)^3 + 8] \div (-4) \\ &= -[-64 + 8] \div (-4) \\ &= -(-56) \div (-4) \\ &= 56 \div (-4) \\ &= -14 \end{aligned}$$

33) **D** In the bottom two rectangles, the GCF of 21 and 30 is 3, so they share a height of 3 cm, and their respective widths are $21 \div 3 = 7$ cm and $30 \div 3 = 10$ cm. Therefore, the width of the larger rectangle is $10 + 7 = 17$ cm. The area of the top left rectangle is given as 39 cm^2. The factor set of 39 in pairs is: {1, 39, 3, 13}, since the width of the larger rectangle is 17cm, the dimensions of the top left rectangle must be 3 and 13. Therefore, the top two rectangles share a height of 3 cm. The width of the top left rectangle is 13cm. The width of rectangle A is $17 - 13 = 4$ cm. Therefore, the area of rectangle A is $3 \times 4 = 12$ cm^2.



Since the right rectangles of the first three rows share a width of 4 cm, the widths of the right rectangles of second and third rows are $16 \div 4 = 4$ cm and $8 \div 4 = 2$ cm respectively. Therefore, the dimensions of rectangle B are 13 and 4 and its area is $13 \times 4 = 52$ cm^2.

For the third row, since the height is 2 cm, width of the middle rectangle is $12 \div 2 = 6$ cm. So, the width of rectangle C is $17 - 4 - 6 = 7$ cm. Its dimensions are 7 and 2. Therefore, area of rectangle C is $7 \times 2 = 14$ cm^2.

Total area of the shaded rectangles A, B, and C is $= 12 + 52 + 14$

$= 78$ cm^2.

34) **B** If m, n, and p have exactly two factors then they are prime numbers. The powers of 2 less than 100 are: 2, 4, 8, 16, 32, and 64. The only prime numbers less than 100 that precede one of the listed powers of 2 are 3, 7, and 31. As the numbers 3, 7, and 31 are prime numbers, the LCM (3, 7, 31) = 3 × 7 × 31

= 651

35) **C** Since Emily takes 6 hours to build the castle, she would build $\frac{1}{6}$ of the castle in 1 hour. In four hours, while working with her brother Marco, Emily would build $4 \times \frac{1}{6} = \frac{2}{3}$ of the castle. Therefore, in four hours, Marco would build $1 - \frac{2}{3} = \frac{1}{3}$ of the castle. Thus, to build the whole castle when working alone, Marco would need 4 × 3 = 12 hours.

36) **A** Try different options until the value of the expression equals 5.

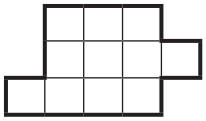
1 $\boxed{-}$ 6 $\boxed{\times}$ (–2) $\boxed{\div}$ 3 = 1 – (–12) ÷ 3

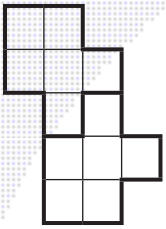
= 1 + 4

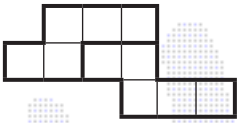
= 5

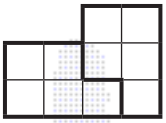
The operation that was not used is addition.

37) **A** As indicated in the diagram below, the figures in options B), C), and D) can be cut along the grid lines to make exactly two identical (congruent) shapes. Therefore, the figure in option A) cannot be divided into two congruent shapes. Also, notice that this figure is the only figure which is made of an odd number of small squares and, therefore cannot be divided into two shapes with the same number of small squares.

A) 

B) 

C) 

D) 

38) **A** Using "MATH" as the keyword and "E" as the key letter, the encryption of the alphabet would be as follows:

A	B	C	D	**E**	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
W	X	Y	Z	**M**	**A**	**T**	**H**	B	C	D	E	F	G	I	J	K	L	N	O	P	Q	R	S	U	V

Therefore, S = N, U = P, R = L, P = J, I = B, and E = M. Thus, the word "SURPRISE" would encrypt into "NPLJLBNM".

39) **B** Make an organized chart to determine who joins which clubs. It is known that Natalie studies Latin and Jaia is not a member of the drama club. It is also known that the member of the math club studies Japanese. Since Natalie does not study Japanese, she is not in the math club. There are two remaining options: either Maya or Jaia are in the math club and studying Japanese.

Let's assume Maya is in the math club and studying Japanese (see the check marks in the shaded cells in the first logic table). Then, Jaia would be a member of the art club and she would study French, but it is known that the girl who studies French is not in the art club. This option is not correct.

	Math	Art	Drama	French	Japanese	Latin
Natalie	X			X	X	✓
Maya	✓				✓	X
Jaia			X			X

Hence, Jaia is in the math club and studying Japanese (see the check marks in the shaded cells in the second logic table). Maya must therefore study French and be a member of the drama club, which leaves Natalie as the member of the art club.

	Math	Art	Drama	French	Japanese	Latin
Natalie	X	✓	X	X	X	✓
Maya	X	X	✓	✓	X	X
Jaia	✓	X	X	X	✓	X

40) **B** The pattern is to use the three corner numbers to produce the value inside the triangle. Double the sum of the squares of the corner numbers to get the value of the inside number. For example, in the first triangle, $2 \times (3^2 + 2^2 + 7^2) = 2 \times 62$

$= 124$

Thus, the number in the last triangle is $2 \times (5^2 + 2^2 + 6^2) = 2 \times 65$

$= 130$